Moments d 'inerties et produit d'inertie des sections

1: Triangle:

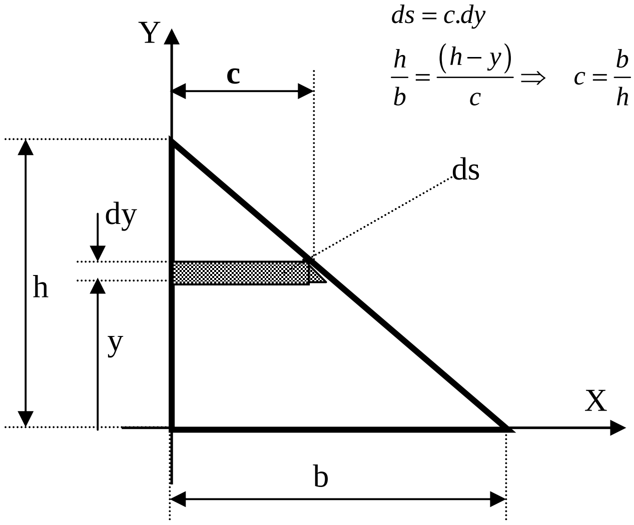
$$I_x = \int_s y^2 \cdot ds = \frac{b}{h}\int_0^h y^2 \left(h-y\right) dy$$

$$= \frac{b}{h}\int_0^h \left(y^2 h - y^3\right) dy = \frac{b}{h}\left[h\frac{y^3}{3} - \frac{y^4}{4}\right]_0^h = \frac{bh^3}{12}$$

$$I_{Gx} = \frac{bh^3}{12} - \left(\frac{h}{3}\right)^2 \cdot \frac{bh}{2} = \frac{bh^3}{36}$$

$$ds = c.dy$$

$$\frac{h}{b} = \frac{\left(h-y\right)}{c} \Rightarrow \quad c = \frac{b}{h}.\left(h-y\right)$$



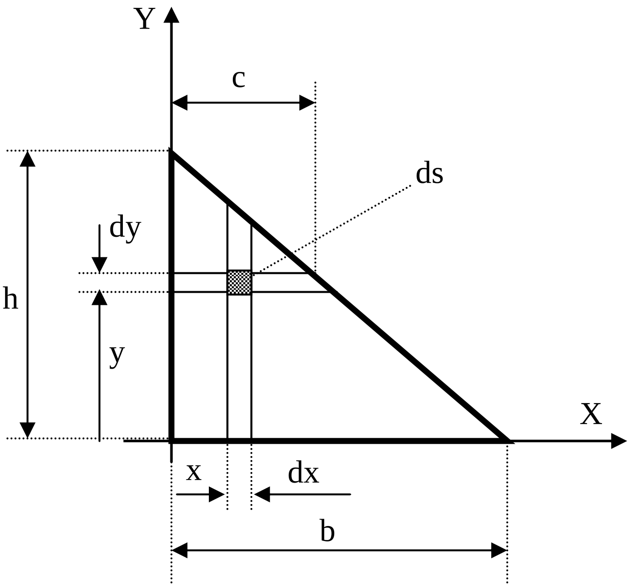
Produit d'inertie:

$$I_{xy} = \int_s xyds = \int_0^h ydy \int_0^c xdx$$

$$I_{xy} = \int_0^h ydy \cdot \frac{c^2}{2} = \frac{b^2}{2h^2}\int_0^h y\left(h-y\right)^2 dy$$

$$I_{xy} = \frac{b^2}{2h^2}\left[\int_0^h \left(yh^2 + y^3 - 2hy^2\right) dy\right] = \frac{b^2h^2}{24}$$

$$I_{GxGy} = \frac{b^2h^2}{24} - \left(\frac{h}{3}\right)\left(\frac{b}{3}\right)\left(\frac{bh}{2}\right) = \frac{b^2h^2}{24} - \frac{b^2h^2}{18} = -\frac{b^2h^2}{72}$$

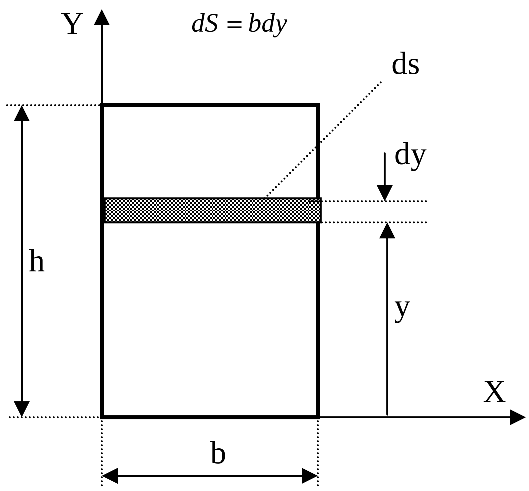


2: Rectangle:

$$I_x = \int_s y^2 ds = b\int_0^h y^2 dy = b\left[\frac{y^3}{3}\right]_0^h = \frac{bh^3}{3}$$

$$I_{Gx} = \frac{bh^3}{3} - \left(\frac{h}{2}\right)^2 bh = \frac{bh^3}{12}$$

$$dS = bdy$$



3: SURFACE CIRCULAIRE

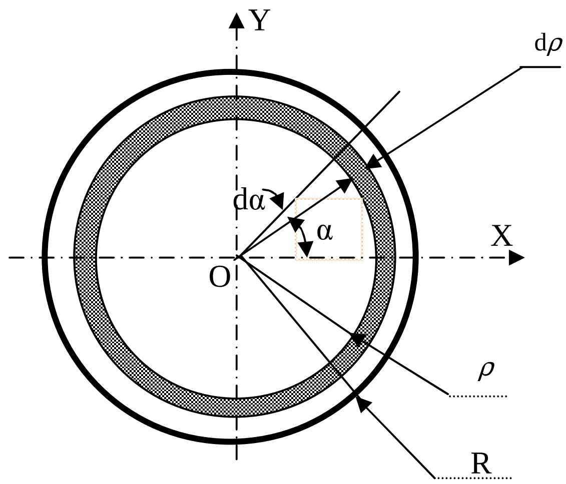
$$I_x = \int_s y^2 ds = \int_0^R \int_0^\alpha \rho^3 \sin^2 \alpha d\rho d\alpha$$

$$I_x = \int_0^R \rho^3 d\rho \int_0^\alpha \sin^2 \alpha d\alpha = \frac{R^4}{4} \int_0^\alpha \sin^2 \alpha d\alpha$$

ds = $\rho d\alpha d\rho$

y = $\rho \sin\alpha$

x = $\rho \cos\alpha$



a) Si **α = 2π** => $I_x = I_y = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$

$$I_x = I_y = I_{Gx} = I_{Gy} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$$

b) Si **α = π** => $I_x = I_y = \frac{\pi R^4}{8} = \frac{\pi D^4}{128}$

- $I_{Gx} = I_x - \left(\frac{4R}{3\pi}\right)^2 \frac{\pi R^2}{2} = \frac{\pi R^4}{8} - \frac{8\pi R^4}{9}$
- $I_{Gy} = I_y = \frac{\pi R^4}{8} = \frac{\pi D^4}{128}$

c) Si **α = π /2** => $I_x = I_y = \frac{\pi R^4}{16} = \frac{\pi D^4}{256}$

$$I_{Gx} = I_{Gy} = \frac{\pi R^4}{16} - \left(\frac{4R}{3\pi}\right)^2 \frac{\pi R^2}{4} = \frac{\pi R^4}{16} - \frac{4\pi R^4}{9}$$

Produit d'inertie:

Formule trigonométrique:

$$2\sin\alpha\cos\alpha = \sin 2\alpha$$

$$\int \sin 2\alpha \, d\alpha = -\frac{1}{2}\cos 2\alpha$$

$$Ixy = \int_s \rho^3 \sin\alpha \cos\alpha d\alpha d\rho = \frac{1}{2}\left[\frac{\rho^4}{4}\right]_0^R \int_0^\alpha \sin 2\alpha d\alpha$$

$$Ixy = -\frac{R^4}{16}\left[\cos 2\alpha\right]_0^\alpha$$

$$si \begin{cases} \alpha = 0 \quad \Rightarrow Ixy = -\frac{R^4}{16}[1-1] = 0 \\ \alpha = \frac{\pi}{2} \quad \Rightarrow Ixy = -\frac{R^4}{16}[0-1] = \frac{R^4}{16} \\ \alpha = \pi \quad \Rightarrow Ixy = -\frac{R^4}{16}[1-1] = 0 \end{cases}$$

Remarque: L'étudiant est censé de savoir déterminer les moments d'inerties de n'importe quelle section.